

# Errata for Differential Geometry of Curves and Surfaces

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The following list reflects errata found in the text as of August 10, 2011.

## Chapter 1

**Page 3 line 10:**  $t \in [0, 2\pi]$  should be  $t \in [0, 2\pi/\omega]$ .

**Page 9 line -1:** The distance formula in the display equation should be

$$\sqrt{\|\vec{p} - \vec{b}\|^2 - \frac{(\vec{a} \cdot (\vec{p} - \vec{b}))^2}{\|\vec{a}\|^2}} = \frac{\|\vec{a} \times (\vec{p} - \vec{b})\|}{\|\vec{a}\|},$$

**Page 29 line 23:** Definition 1.4.4, in the last line “of order 2.” should be “of order at least 2.”

**Page 36 line 1:** “Given a function  $\kappa_g(s)$ ,” should be “Given a piecewise continuous function  $\kappa_g(s)$ ,”. (This is necessary to ensure that  $\kappa_g(s)$  is integrable and that  $\cos \theta(s)$  and  $\sin \theta(s)$  are integrable.)

**Page 38 line 7:** In Problem 1.5.2,  $\vec{x}'(s) = (1, 0)$  should be the initial condition  $\vec{x}'(0) = (1, 0)$ .

## Chapter 2

**Page 41 line 9:** In the assumptions on  $\mathcal{C}$  we must also assume that  $\mathcal{C}$  is positively oriented.

**Page 53 line 21:** In Section 2.4, we assume that all curves can be parametrized with a parametrization of class  $C^2$ .

## Chapter 3

**Page 70 line 8:** “parametrized curve” should be “parametrized, regular curve.”

**Page 80 line 3-5:** The interpretation of the sign of torsion is reversed. We have  $\tau(t_0) > 0$  at  $\vec{x}(t_0)$  when the curve comes up through the osculating plane and we have  $\tau(t_0) < 0$  at  $\vec{x}(t_0)$  when the curve goes down through the osculating plane. (Figure 3.5 is correct.)

**Page 81 line -1:** “contact of order 3” should be “contact of order at least 3.”

## Chapter 4

**Page 88 line 10:** In Stokes’s Theorem, we must also assume that  $\mathcal{C}$  is oriented according to the right hand rule with respect to the oriented surface  $\mathcal{S}$ .

**Page 91 line -4:**  $AB = 2 \sin\left(\frac{AB}{2}\right)$  should be  $AB = 2 \sin\left(\frac{\overline{AB}}{2}\right)$ .

**Page 91 line -3:** “Let  $\Gamma$  be a regular curve” should be “Let  $\Gamma$  be a closed regular curve”.

**Page 92 line 4:** “ $L \leq 2\pi$ ” should be “ $L < 2\pi$ ”.

**Page 97 line 12:** “ $\vec{X}(t)$ ” should be “ $\vec{x}(t)$ ”.

**Page 102 line -2:** “in Figure 4.6” should be “in the oriented diagram on the right in Figure 4.6”

## Chapter 5

**Page 114 line 13:** “the  $x$ -axis” should be “the  $y$ -axis.”

**Page 120 line 7:** The indices  $m$  and  $n$  in the matrix of partial derivatives are reversed.

**Page 121 line 5:** “ $dF_q$  does not have maximal rank” should be “ $dF_q$  does not exist or does not have maximal rank”.

**Page 121 line -8:** “ $U \in \mathbb{R}^2$ ” should be “ $U \subseteq \mathbb{R}^2$ ”.

**Page 123 line -6:** “ $|x \geq 0$  if  $y = 0$ ” should be “ $|x \geq 0$  and  $y = 0$ ”.

**Page 124 line 3:** “ $|x \leq 0$  if  $z = 0$ ” should be “ $|x \leq 0$  and  $z = 0$ ”.

**Page 131 line 6:**  $F(U) = U'$  should be  $F(U') = U$ .

**Page 133 line 2:**  $dF_q$  should be  $dF_{q'}$ .

**Page 137 line -2:** “for any fixed  $u_0 \in I$ , along any curve  $\vec{X}(u_0, v)$ ,” should read “for any fixed  $t_0 \in I$ , along any curve  $\vec{X}(t_0, u)$ ,”

**Page 138 line 18:** “is a regular curve” should be “is a disjoint union of regular curves”

## Chapter 6

**Page 160 line 10:** “two-dimensional subspace of  $\mathbb{R}^2$ ” should be “two-dimensional subspace of  $\mathbb{R}^3$ ”

**Page 169 line 3:**  $2L_{12}st$  should be  $2L_{12}(q)st$ .

**Page 169 line 10:**  $+\vec{X}_{vv}(u_0, v_0)(v - v_0)^2$  should be  $+\frac{1}{2}\vec{X}_{vv}(u_0, v_0)(v - v_0)^2$ .

**Page 174 line -3:** The display equation should read:

$$-L_{ij} = \vec{N}_i \cdot \vec{X}_j = \left( \sum_{k=1}^2 a_i^k \vec{X}_k \right) \cdot \vec{X}_j,$$

**Page 175 line 2,7:** The author incorrectly listed the transpose of the matrix of the differential of the Gauss map instead of the matrix itself. Equation (6.25) should read

$$-\begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} a_1^1 & a_2^1 \\ a_1^2 & a_2^2 \end{pmatrix}.$$

Consequently, Equation (6.26), which defines the Weingarten equations should read

$$\begin{pmatrix} a_1^1 & a_2^1 \\ a_1^2 & a_2^2 \end{pmatrix} = - \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}^{-1} \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix}.$$

**Page 179 line 14,15:**  $Lg^{-1}$  in both of those lines should be  $g^{-1}L$ .

**Page 180 line -1:** Because of the previous comment, the matrix  $(a_i^j)$  is the transpose of what is given in the text.

**Page 181 line -5:** Equation (6.30) should be

$$\begin{pmatrix} v' \\ -u' \end{pmatrix} \cdot dn_p \begin{pmatrix} u' \\ v' \end{pmatrix}.$$

**Page 181 line -4:** “Any curve” should be “Any regular curve”

**Page 182 line -7:** The display equation should be

$$L_{11}g_{12} = L_{12}g_{11} \quad L_{11}g_{22} = L_{22}g_{11} \quad L_{22}g_{12} = L_{12}g_{22}.$$

**Page 183 line 8:**  $-Lg^{-1}$  should be  $-g^{-1}L$ .

**Page 183 line -3:** “ $L_{21}g_{11} = 0$  or  $L_{12}g_{22} = 0$ ” should be “ $L_{21}g_{11} = 0$  and  $L_{12}g_{22} = 0$ ”.

**Page 186 line 22:** Delete the  $\vec{T}(t)$  in the last parentheses.

Page 187 line 2: The display equation should read:

$$L_{11} = \frac{-fh'}{\sqrt{f'(v)^2 + h'(v)^2}}, \quad L_{12} = L_{21} = 0, \quad L_{22} = \frac{f''h' - f'h''}{\sqrt{f'(v)^2 + h'(v)^2}}.$$

Page 187 line 6: The display equation should read:

$$\kappa_1(u, v) = -\frac{h'(v)}{f(v)\sqrt{f'(v)^2 + h'(v)^2}}, \quad \kappa_2(u, v) = \frac{f''(v)h'(v) - f'(v)h''(v)}{((f'(v))^2 + (h'(v))^2)^{3/2}},$$

Page 189 line 6: “regular surface” should be “regular oriented surface”

Page 189 line -9: “mean curvature of  $S$ ” should be “mean curvature (up to sign) of  $S$ ”

Page 191 line 10:  $-Lg^{-1}$  should be  $-g^{-1}L$ .

Page 192 line 19: The  $L_{ij}$  matrix is off by a sign.

Page 198 line 14: “a curve  $C$ ” should be “a simple, closed curve  $C$ ”

Page 201 line 9:  $\vec{\beta}' \times \vec{w} = (\vec{\beta}' \times \vec{w}) \cdot \vec{w}' / \|\vec{w}'\|$  should be  $\|\vec{\beta}' \times \vec{w}\| = |(\vec{\beta}' \times \vec{w}) \cdot \vec{w}'| / \|\vec{w}'\|$

Page 201 line 18: Equation (6.44) should read

$$(g_{ij}) = \begin{pmatrix} \|\vec{\beta}'\|^2 + u^2\|\vec{w}'\|^2 & \vec{\beta}' \cdot \vec{w}' \\ \vec{\beta}' \cdot \vec{w}' & 1 \end{pmatrix}$$

Page 202 line 20-21: “is a cone if and only if  $\vec{\alpha}'(t) = 0$ ” should be “is a cone if  $\vec{\alpha}'(t) = 0$ ”

Page 204 line 11:  $D' \in U$  should be  $D' \subseteq U$ .

Page 205 line 8:  $\|\vec{X}_u \times \vec{X}_v\| \neq 0$  should be  $\|\vec{X}_u \times \vec{X}_v\| \neq 0$

Page 207 line 10: In Problem 6.6.10, the beginning “Show that” should read “Prove or disprove that”.

Page 207 line 14-15: “parametrized by  $\vec{Z}_t(u, v) = (1-t)\vec{X}(u, v) + t\vec{Y}(u, v)$ ” should be “parametrized by  $\vec{Z}^t(u, v) = (1-t)\vec{X}(u, v) + t\vec{Y}(u, v)$ ”

## Chapter 7

Page 216 line -3: Remove the comma in the display equation.

Page 222 line -3: The display equation should read

$$a_j^i = -g^{ik}L_{kj}.$$

Page 244 line -9: The display equation for the parametrization of the torus should be

$$\vec{X}(u, v) = ((b + a \cos v) \cos u, (b + a \cos v) \sin u, a \sin v),$$

Page 244 line -2,-1: “Problem 6.5.5” should be “Example 6.5.5.”

Page 255 line -4: “18 partial differential equation” should be “18 partial differential equations”

## Chapter 8

Page 281 line 6: “direction of  $\vec{v}$ .” should “direction of  $\vec{u}$ .”

Page 281 line 14:  $F : (R)^n \rightarrow \mathbb{R}^n$  should be  $F : \mathbb{R}^n \rightarrow \mathbb{R}^n$ .

Page 298 line -6: In the definition of piecewise regular, one assumes that  $\vec{\alpha}'(t)$  has nonzero one-sided limits at each  $t_i$  but such that the one-sided limits are distinct.

**Page 299 line 5:** “collinear” should be “collinear and pointing in opposite directions”.

**Page 300 line 2:** “orange” should be “green”

**Page 300 line 16:** Example 8.1.9 should be Example 8.4.4

**Page 303 line 18:** “that it consider a” should be “that it considers a”.

**Page 310 line 4:** “independedent” should be “independent”

**Page 314 line 5:** The display equation should read

$$\iint_S K dS > 0.$$

**Page 314 line -6:** “propositions through from 23 definitions” should be “propositions from 23 definitions”

**page 315 line -1:** “piecewise regular, simple, closed curve.” should be “piecewise regular, simple, positively oriented, closed curve.”